

## LEPTON TRANSMUTATIONS FROM A ROTATING MASS MATRIX

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Fermion mass matrices generally rotate in generation space under scale changes, which can lead to fermions of different generations transmuting into one another. The effect is examined in detail and its cross-section calculated for  $\gamma + \ell_\alpha \rightarrow \gamma + \ell_\beta$  with  $\ell_\alpha \neq \ell_\beta$  the charged leptons  $e, \mu$ , or  $\tau$ . For the (conventional) Standard Model, this is weak and probably undetectable, though with some notable exceptions. But for the Dualized Standard Model, which we advocate and have already used quite successfully to explain quark mixing and neutrino oscillation, the effect is larger and could be observable. Estimates of transmutational decays are also given.

## 1 Introduction

By a rotating mass matrix we mean one which undergoes unitary transformations through scale changes, as a result of the renormalization group equation, as can happen in many gauge theories. This means that even if the mass matrix  $m$  is diagonal (in generation space) at a certain scale, it will not remain so as the scale changes. Hence in general we can expect nonzero transition between fermions of different generations:

$$\ell_\alpha \rightarrow \ell_\beta, \quad \alpha \neq \beta, \quad (1)$$

such as  $e \rightarrow \mu$ ,  $e \rightarrow \tau$ ,  $\mu \rightarrow \tau$ . We shall use the term 'transmutation' for this direct transition to distinguish it from e.g.  $e \rightarrow \mu$  conversion via FCNC.

Here I shall concentrate on transmutational processes<sup>1,2</sup> in the standard model (SM) and, in greater detail, in the dualized standard model (DSM).

## 2 Mass matrix rotation

In the SM, because the leptonic MNS mixing matrix<sup>3</sup>  $U$  is nontrivial<sup>4</sup>, the mass matrix  $L$  for the charged leptons will rotate as a result of the following term in the linearized RGE<sup>5</sup>:

$$\frac{dL}{d\mu} = \frac{3}{128\pi^2} \frac{1}{246^2} (ULU^\dagger)(ULU^\dagger)^\dagger L + \dots, \quad (2)$$

where  $ULU^\dagger = N$  the neutrino (Dirac) mass matrix. Therefore  $L$  cannot be diagonal at all scales. The magnitude of the off-diagonal elements will depend on poorly known or unknown quantities such as the mixing  $U$  and the Dirac mass  $m_3$  of the heaviest neutrino. If we take the present popular theoretical biases, namely that  $U$  is bimaximal<sup>6</sup> and that  $m_3$  is around the  $t$  quark mass, then (2) gives

$$\langle \mu | \tau \rangle \text{ changes by } \sim 5.5 \times 10^{-3} \text{ GeV}$$

$$\langle e | \tau \rangle \text{ changes by } \sim 1.8 \times 10^{-7} \text{ GeV}$$

$$\langle e | \mu \rangle \text{ changes by } \sim 1.1 \times 10^{-8} \text{ GeV}$$

per decade change in energy.

In the DSM<sup>7</sup>, the fermion mass matrix is of the following factorized form:

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), \quad (3)$$

where  $m_T$  is essentially the mass of the heaviest generation. Under renormalization  $m$  remains factorized, but the vector  $(x, y, z)$  changes as

$$\frac{d}{d\mu} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{5}{32\pi^2} \rho^2 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \quad (4)$$

where  $\rho$  is a (fitted) constant and

$$x_1 = \frac{x(x^2 - y^2)}{x^2 + y^2} + \frac{x(x^2 - z^2)}{x^2 + z^2}, \quad \text{cyclic.} \quad (5)$$

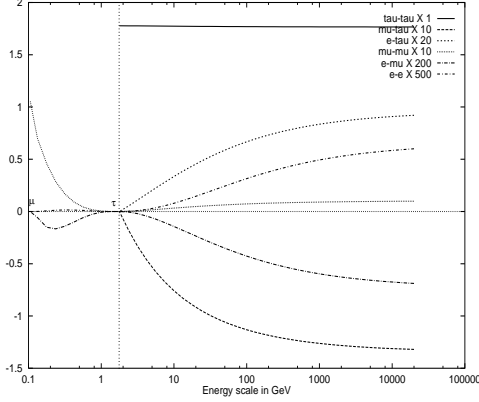


Figure 1. Elements of the rotating mass matrix in GeV for charged leptons in the DSM scheme.

The off-diagonal elements have been calculated explicitly, using 3 free parameters determined by fitting experimental mass and mixing parameters (giving sensible predictions for the remaining parameters)<sup>8</sup>. These are shown in Figure 1. Hence the results we report below are entirely parameter-free.

### 3 Lepton states

To define lepton states one must diagonalize the mass matrix, but since the eigenvectors depend on scale, there is no canonical recipe.

We suggest two quite different schemes for exploration: fixed scale diagonalization (FSD) which is applicable to SM, and step-by-step diagonalization (SSD) which is an inherent part of DSM<sup>7,1,2</sup>.

### 4 Transmutational decays

Using the results of §2 we give estimates for the branching ratios of transmutational decays<sup>1</sup>. With some exceptions, SM (with FSD) estimates are all far below present experimental bounds and are hence not so interesting. One exception is the process  $\mu^- \rightarrow e^- e^+ e^-$ , where one could get a branching ratio of  $10^{-3}$  (experimental limit  $10^{-12}$ ), if one applied FSD naively. This shows how sensitive these calculations are to transmutational

Table 1. Branching ratios of transmutational decays.

Decays	DSM est.	Expt limit
$Z^0 \rightarrow \tau^- \mu^+$	$4 \times 10^{-8}$	$1.2 \times 10^{-5}$
$\pi^0 \rightarrow \mu^- e^+$	$3 \times 10^{-9}$	$1.7 \times 10^{-8}$
$\psi \rightarrow \mu^+ \tau^-$	$6 \times 10^{-6}$	not given
$\Upsilon \rightarrow \mu^+ \tau^-$	$2 \times 10^{-6}$	not given
$\mu^- \rightarrow e^- \gamma$	0	$4.9 \times 10^{-11}$
$\mu^- \rightarrow e^- e^+ e^-$	0	$1.0 \times 10^{-12}$

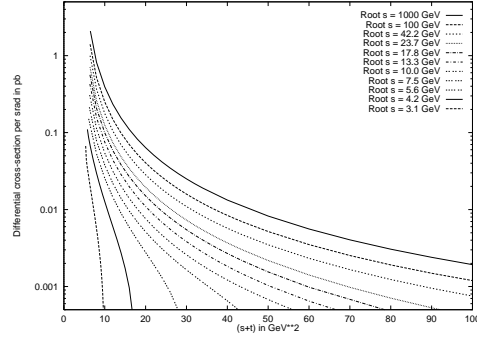


Figure 2. Differential cross sections for  $\gamma e \rightarrow \gamma \tau$ .

model and/or diagonalization scheme.

The parameter-free estimates in DSM give branching ratios which are in general larger but still below present experimental limits, as indicated in Table 1. It is important to note that because of SSD the branching ratios of transmutational leptonic decays are automatically zero to first order (Figure 1). The  $\pi^0$  decay is of particular interest as being less than one order from the experimental bound.

### 5 Photo-transmutation

We studied<sup>2</sup> (mainly for DSM) the following

$$\gamma + \ell_\alpha \longrightarrow \gamma + \ell_\beta, \quad \alpha \neq \beta. \quad (6)$$

We calculated the cross sections for:  $\gamma e \rightarrow \gamma \mu$ ,  $\gamma e \rightarrow \gamma \tau$ ,  $\gamma \mu \rightarrow \gamma \tau$ , leaving out  $\tau$ -initiated reactions as being experimentally unrealistic at present. A sample of the DSM

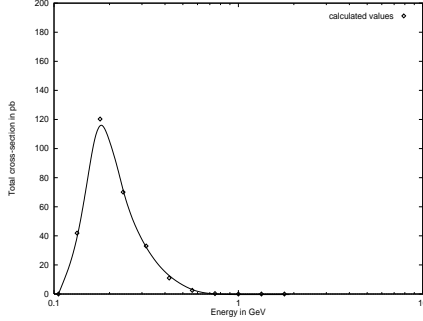


Figure 3. Total cross section for  $\gamma e \rightarrow \gamma \mu$ .

results is presented in Figure 2, for a range of c.m. energies  $\sqrt{s}$ . Because of the calculated form of the rotation matrix, we get in general:  $\gamma \mu \rightarrow \gamma \tau > \gamma e \rightarrow \gamma \tau > \gamma e \rightarrow \gamma \mu$ . However, at low energies  $\gamma e \rightarrow \gamma \mu$  becomes quite sizeable, as seen in Figure 3, where the total cross section has a peak of  $\sim 100$  pb at c.m. energy  $\sim 200$  MeV.

SM calculations depend on further assumptions, under which e.g.  $\mu \rightarrow \tau$  at  $\sqrt{s} = 17.8$  GeV is  $\sim 3$  orders smaller than DSM.

## 6 Possible experimental tests

The DSM predictions for the transmutational decay modes:  $\pi^0 \rightarrow \mu^- e^+$ ,  $\psi \rightarrow \tau^- \mu^+$ ,  $\Upsilon \rightarrow \tau^- \mu^+$  could be near experimental limits and sensitivities, for LEP, BEPC and B-factories.

For photo-transmutations, one may consider virtual  $\gamma$  from  $e^+e^-$  colliders. Above  $\tau$ , it is more profitable to look for<sup>9</sup>  $e \rightarrow \tau$ . Below  $\tau$ , it is more profitable to look for  $e \rightarrow \mu$ , at around 200 MeV c.m. energy. Again, LEP and/or BEPC may provide tests.

## 7 Conclusions

- Lepton transmutation is a necessary consequence in both SM and DSM.
- The SM results are in general smaller than DSM results, but there are uncertainties and further assumptions.
- The DSM calculations are entirely

parameter-free. There are no violations of data in all the cases we were able to consider.

- Experimental tests of mainly DSM predictions seem feasible in the near future, for the following:
  - decays of  $\pi^0, \psi, \Upsilon$ ,
  - photo-transmutation of  $e \rightarrow \tau$  at high and  $e \rightarrow \mu$  at low energy,
  - other processes e.g.  $e^+e^- \rightarrow e^+\mu^-$ .
- Whether our results will pass these tests or not, there is exciting new physics that has to be explored.

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